Short notes on Discrete Systems

Hasan Ayaz
School of Biomedical Engineering Science and Health Systems,
Drexel University

A system can be defined as a set of rules that create output signal from input signal. Discrete time system is a system that works with discrete time input and output signals. Input signals or excitations are generally shown with x, and output signal or response, are usually shown with y. Thus a system can be shown by $x(n) \rightarrow y(n)$ where n is the discrete time index. Discrete time systems can be classified according to general properties they satisfy. Main properties and types of discrete systems are discussed briefly below.

First property of concern is time. If the input-output relationship of a system does not change with time, the system is called time-invariant. For a time-invariant system, if the output to a certain input function is known, the output to the time-shifted version of this input can also be known. It would be simply, the time shifted version of the initial output. $x(n) \rightarrow y(n)$ then $x(n-k) \rightarrow y(n-k)$ where k is the shift

If the output signal of system depends on only the current input signal, the system is called memoryless. Otherwise, if the output depends on past or future values, system is said to have memory or called dynamic.

Another property of a system is linearity. A system is linear if it satisfies both additivity and scaling properties. Otherwise, system is called nonlinear. In other words, if
$x_1(n) \rightarrow y_1(n)$ and $x_2(n) \rightarrow y_2(n)$ than
$Ax_1(n) + Bx_2(n) \rightarrow Ay_1(n) + By_2(n)$ where A and B are arbitrary constants

Another property of discrete systems is causality. If the output of the system depends only present and past values of the input, that system is called causal system. However, if there's a dependency on future values, the system is called non-causal.

Obviously, in our physical world, it is impossible to observe future values, so non-causal systems can not be physically realizable. But, they can exist as an algorithm, on computer, when we have all input signal before starting processing.

Last but not least, stability of the system can be assessed by bounded input. A system is called BIBO (bounded input bounded output) stable if all bounded inputs create bounded output. If there's an M such that $|x(n)| \leq M < \infty$ that there's an N such that $|y(n)| \leq N < \infty$

Discrete time, linear, time invariant (LTI) systems are the most important class of systems because, when a signal is represented by a weighted sum of unit samples, only if the system is linear and time-invariant, output can also be represented as the weights of the same unit sample sequence.

A LTI system can be represented in general form as follows:

$y(n) = \sum_{k=0}^{M} A_k x(n-k) - \sum_{k=1}^{N} B_k y(n-k)$ where A_k and B_k are system defining constants.

Since any input arbitrary input signal can be decomposed into weighted impulses, knowing the response to a single impulse would reveal the response to the arbitrary response if the system is LTI. Thus, an LTI system is characterized by its impulse response shown by h(n). Impulse response satisfies the following equation:

$$y(n) = \sum_{k=0}^{\infty} h(k) x(n-k)$$

which states that the convolution of impulse response and input signal gives the output signal.

LTI systems can be classified by their impulse response. If the impulse response has finite length, or duration, this system is called FIR (Finite-duration Impulse Response). Otherwise, the impulse response is infinite-duration impulse response (IIR). For causal systems all values of impulse response for $n<0$ are 0. When the system is casual and FIR than all values are zero for $n>M$ where $(M+1)$ is the finite length of the impulse response. The practical importance of casual FIR appears here that since $h(n)=0$ for $n<0$ and $n>M$ the convolution above simplifies to

$y(n) = \sum_{k=0}^{M} h(k) x(n-k)$ means that, we have to do only M+1 many multiplications to find the output instead of infinite.