Short Notes on Design of Digital Filters for Discrete Systems

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Digital filters are discrete systems that operate on a discrete signal to create an output signal. An introduction to discrete systems and used notation is given in a previous document on the same webpage. Frequency selective digital filter characteristics are defined in frequency domain and in return filter coefficients found by filter design. Filter coefficients are merely the impulse response of the filter. For a casual FIR system:

$y(n) = \sum_{k=0}^{M} A_k x(n-k)$ where A_k is a set of numbers or finite-duration impulse response. If the set of numbers A_k represents a symmetry with respect to middle element, then the filter will have linear phase. Linear phase is required for most applications. Our focus is on designing and applying linear phase stable filters. Finally design of filters is to calculate the first half of the set of coefficients (impulse response) since other half is simply the reverse ordered of the first half.

Design of linear phase casual FIR filters with windows starts with specifying the desired ideal frequency response $H_d(w)$. Since $H_d(w)$ is ideal, its inverse Fourier transform, $h_d(n)$, has infinite duration in time domain. $h_d(n)$ is calculated from $H_d(w)$ by applying inverse Fourier transform as follows:

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(w) e^{jwn} dw \qquad (1)$$

This infinite length $h_d(n)$ function is truncated to fit in a certain window. To do, window function is multiplied with desired response in time domain.

$$h(n) = h_d(n) w(n) \qquad (2)$$

where w(n) is zero outside the interval [0,M] and it is symmetric around its midpoint. After unit sample is windowed, the filter impulse response is left. The choice of the window function affects the overall filter characteristics.

Rectangular Window

Rectangular window is analytically defined as in equation 3. The time domain and frequency domain characteristics are given in Figure 1 for filter order of 20. The graph is obtained in Matlab. As seen in the time domain graph, the window is symmetric around its midpoint.

$$w(n) = \begin{cases} 1, & 0 \le n \le M \\ 0, & otherwise \end{cases} \qquad (3)$$

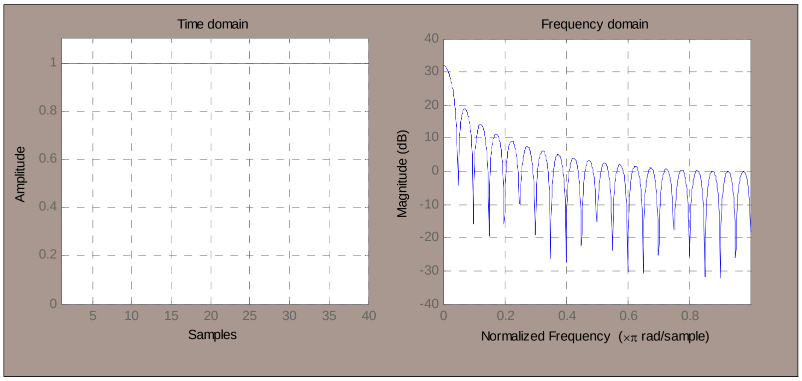

Figure 1 Rectangular window (order=40)

Blackman Window

Blackman window is analytically defined as in equation 4. The time domain and frequency domain characteristics are given in Figure 2 for filter order of 40. The graph is obtained in Matlab. As seen in the time domain graph, the window is symmetric around its midpoint.

$$w(n)=\begin{cases}0.42-0.5\cos(\frac{2\pi n}{M})+0.08\cos(\frac{4\pi n}{M}), & 0\le n\le M\\ 0, & otherwise\end{cases} \qquad (4)$$

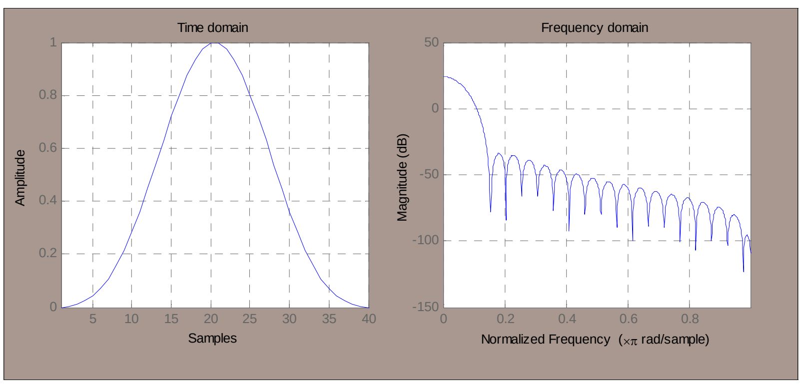

Figure 2 Blackman window (order=40)

Hamming Window

Hamming window is analytically defined as in equation 5. The time domain and frequency domain characteristics are given in Figure 3 for filter order of 40. The graph is obtained in Matlab. As seen in the time domain graph, the window is symmetric around its midpoint.

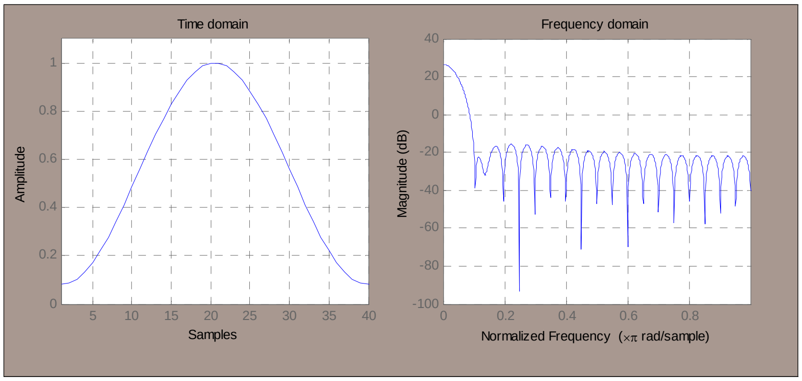

Figure 3 Hamming window (Order=40)

$$w(n)=\begin{cases}0.54-0.46\cos(\frac{2\pi n}{M}), & 0\le n\le M\\ 0, & otherwise\end{cases} \tag{5}$$

Hanning Window

Hanning (or also called Hann) window is analytically defined as in equation 6. The time domain and frequency domain characteristics are given in Figure 4 for filter order of 40. The graphs are obtained in Matlab. As seen in the time domain graph, the window is symmetric around its midpoint.

$$w(n)=\begin{cases}0.5-0.5\cos(\frac{2\pi n}{M}), & 0\le n\le M\\ 0, & otherwise\end{cases} \tag{6}$$

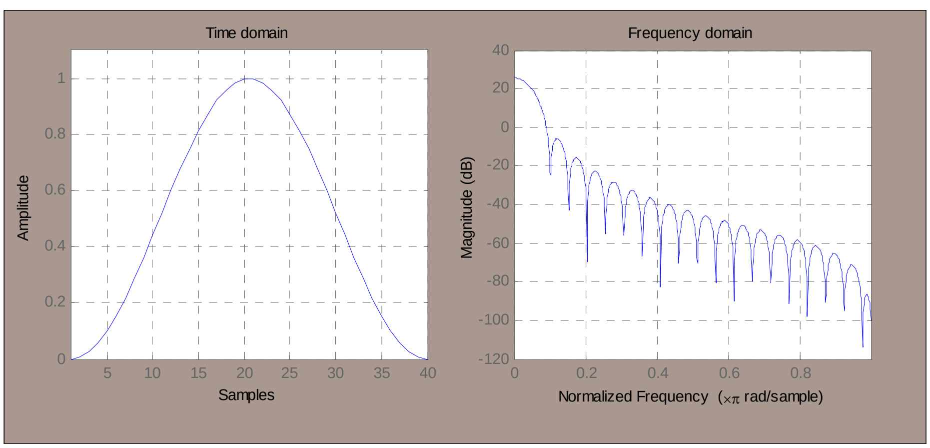

Figure 4 Hannning window (Order=40)